Return to List of Lessons

Calculator Lesson 32

Solving Polynomials

In this lesson we will see two methods for finding the zeros of a polynomial and how to create polynomials with specified zeros.

The first method makes use of the Polynomial solver. To start press RS NUM.SLV DA DA F6-OK. This gets us into the Polynomial Solver. To find the zeros of the polynomial

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

create a vector with the coefficients from a_n to a_0 in the field below “Coefficients.” Be sure to include a zero for any missing coefficients. For example, to find the zeros of the polynomial $x^4 - 15x^2 + 10x + 24$ we must enter the vector of coefficients as [1 0 -15 10 24]. Now, with the field below “Root:” highlighted, press F6-SOLVE and a vector with the zeros [-1 2 3 -4] will show in that field. If you press CANCEL (the ON button) you will see that the vector of zeros has also been placed on the stack.

Why is your author calling these “zeros” when the calculator calls them “roots?” Equations have roots, functions have zeros. The way this information is being entered, it is being treated more like a function than like an equation.

The Polynomial Solver can also be used to create polynomials with a desired set of zeros. Suppose we wish to create a polynomial with -2, 3, 1 + 2i and 1 – 2i as its zeros. Get back into the Polynomial Solver, press DA to highlight the “Roots:” field and enter the vector [-2 3 (1, 2) (1, -2)], make sure the “Coefficients” field is highlighted and press F6-SOLVE. The vector of coefficients [1 -3 1 7 -30] will show in the “Coefficients” field, and will also be placed on the stack.

The Polynomial Solver also has another nice capability. With a vector of coefficients in the “Coefficient” field and that field highlighted press F5-SYMB. Nothing seems to happen, but the polynomial in expanded form has been placed on the stack. With a vector of zeros in the “Roots:” field and that field highlighted, pressing F5-SYMB will place polynomial in factored form on the stack.

The Polynomial Solver also has some unpleasant characteristics. For example, with [1 -5 6] in the “Coefficients” field and that field highlighted, press F5-SYMB then CANCEL. Note that the polynomial is x^2+-5x^1+6x^0. This is not wrong, but it is ugly. To make it pretty, press DA F1-EDIT, then use the arrow key to move the cursor to the right of the offending characters and the backspace key to remove them, so the polynomial becomes x^2-5x+6.

Note that in the Polynomial Solver F1 is an edit command. This puts you into the Matrix Writer to create a vector or to edit an existing vector. However, if we try to use this method to create the vector [-2 3 (1, 2) (1, -2)] we used above, the Matrix Writer gives us an error message. To avoid that we must enter the complex numbers first, then the real numbers. The Matrix Writer does not do that if we go directly into it, but it has problems if we enter through the Polynomial Solver. It is a mystery why that happens.

There are other problems with the polynomial solver. For example, try using it to find the zeros of $x^3 - 3x^2 + 3x - 1$ which has the one zero at $x = 1$ with multiplicity 3. You will get a strange looking solution that is close to the correct answer, but leaves much to be desired. The problem is that the calculator probably uses Newton's Method, or some variation thereof. As we saw in Lesson 11, Newton's Method involves dividing by the derivative of the function in question. If we are seeking a zero of multiplicity greater than one, the derivative is also zero at that point. Thus, as we get close to the solution both the function and its derivative are getting close to zero. Dividing by zero is, of course, impossible, and dividing by numbers close to zero tends to cause serious problems when trying to approximate anything. The following method is therefore suggested if there is reason to believe you are seeing a zero of multiplicity greater than one. NOTE: You can anticipate this problem by graphing the polynomial and its derivative. (See Lesson 8). If the two curves intersect the x-axis at the same place, the zero has multiplicity greater than one.

First we must put the calculator into exact mode by going into the CAS dialog box and making sure that _Numeric and _Approx are both unchecked (See Lesson 1). Now place the algebraic object $x^3 - 3x^2 + 3x - 1$ on the stack. Press RS ALG F3-FACTO, and we will see $(x-1)^3$ as the result. We can now see that the zero is 1 with multiplicity 3. One can create a polynomial with specific zeros by entering it in factored form then pressing F2-EXPAN in the ALG screen.

One might wonder, since we have this factoring tool, why would one ever use the Polynomial Solver. The answer is that there are some polynomials the FACTO command simply can't factor. In that case we have no choice by to use the Polynomial Solver. Try, for example, finding the zeros of $x^4 - 4x^3 + 14x^2 - 20x + 13$. The exact solutions are $1+(1+\sqrt{3})i$, $1-(1+\sqrt{3})i$, $1+(1-\sqrt{3})i$, and $1-(1-\sqrt{3})i$, but you can't find even a reasonable approximation with the FACTO command.

Return to List of Lessons