Return to List of Lessons

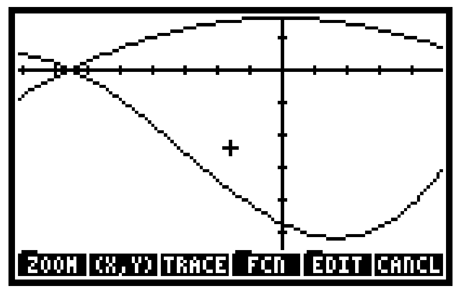
Calculator Lesson 16

Graphs of Areas

In this lesson we will learn a few advanced graphing procedures dealing with functions and areas. For the following examples set the display to Fix 5.

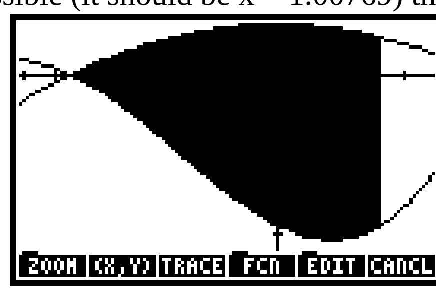
Suppose we wish to find the area bounded by the functions $f(x) = 4 - x^2$ and $g(x) = x^3 + 3x^2 - 4x - 12$ from x = -2 to x = 1. We will graph these functions on the same screen so we see how they are related. In 2D/3D set the Type: to Function and enter

$$\{\, 4 - x^2 \quad x^3 + 3x^2 - 4x - 12 \,\}$$



into EQ:. In WIN set H-View from -2.5 to 1.5 and V-View from -16 to 4, then ERASE and DRAW. You should see the graph shown on the right. Move the cursor to the left near the point where the two curves cross the x-axis. Press F4-FCN then F2-ISECT to verify that the curves cross at the point (2, 0). The coordinates of the intersection will show at the bottom of the screen.

There are two ways that we can mark the area we wish to find. One is to press × to mark the current location of the cursor. Press NXT NXT F6-PICT F2-(X,Y). Move the cursor to the right to get it as close to x = 1 as possible (it should be x = 1.00769) then press NXT > F4-FCN > F5-SHADE. We now see the area we are seeking has been shaded. See the figure to the right. If we now press F4-FCN we see that the F4 key says AREA. It would seem reasonable that if we now pressed that key we would get the area. It doesn't work for two reasons. That key finds the integral of a function between two values of x. In this case if we used × to mark -2 on the x-axis, moved the cursor as close to 1 as we can get it and pressed F4-AREA, we would get 8.93012, which is almost $\int_{-2}^{1}(4 - x^2)dx$ because $(4 - x^2)$ is the first equation in our EQ: list. The reason we do not get the exact value of 9 is that the pixels are not exactly at -2 and 1.



Press CANCE > F5-ERASE > F6-DRAW to redraw the graph. Press F2-(X,Y) and move the cursor to get x as close to 1 as possible. Now move the cursor up to the top curve, press ×, move the cursor to the lower curve and press NXT > F5-EDIT > F3-LINE. This is the other way of marking off the area we are seeking.

We see from the graph that $f(x)$ is the top boundary and $g(x)$ is the bottom between x = -2 and x = 1, so to find the area between these two curve we must compute the integral $\int_{-2}^{1}\left(f(x)-g(x)\right)dx$. To do this using the AREA command, we must graph the function $f(x) - g(x)$ with a H-View width that will make -2 and 1 exact pixel values. To get "nice" pixel values, the H-View width must be $13\times 2^m \times 5^n$ for some integers m and n. We will choose m = -2 and n = 0 for a width of 3.25. In 2D/3D enter the expression $\left(4-x^2\right)-\left(x^3+3x^2-4x-12\right)$. In WIN choose -2.125 to 1.125 for H-View and press F4-AUTO for the V-View, then ERASE and DRAW. Now press F2-(X,Y), move the cursor to x = -2, press ×, move the cursor to x = 1, then press NXT F4-FCN F4-AREA. The result, Area: 33.75000, will show in the lower left corner of the screen. That value will also be on the stack when we return to it. This is the correct value as can be verified by hand or by using the $\int$ command on the calculator (see Lesson 13).

We can see from this example that the graph of the two functions was useful to see the relationship between the functions, and it can be used to create interesting graphics to mark the area, but the $\int$ command is generally a more efficient way to compute the area than the AREA command.

Return to List of Lessons