Supplementary Problems

These problems are intended for more advanced students. Some of them require concepts from Calculus.

1. Use the identity $\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$ to solve each of the following equations:

   (a) $4x^3 - 3x = ½$; (b) $8x^3 - 6x - \sqrt{3} = 0$; (c) $x^3 - 3x + 1 = 0$.

2. Convert each of the following functions into the form $r\sin(\theta - \alpha)$, with appropriate constants r and α, and thus find the maximum and minimum values of $f(\theta)$ without using differentiation. Check using calculus.

   (a) $f(\theta) = 2\sin(\theta) - 2\sqrt{3}\cos(\theta)$;

   (b) $f(\theta) = 20\cos(\theta) + 21\sin(\theta)$.

3. Prove the following identities and generalize:

   (a) $\dfrac{\sin(\theta)}{\sin\left(\frac{\theta}{8}\right)} = 8\cos\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{\theta}{4}\right)\cos\left(\dfrac{\theta}{8}\right)$;

   (b) $\dfrac{\sin(\theta)}{\sin\left(\frac{\theta}{9}\right)} = \left[4\cos^2\left(\dfrac{\theta}{3}\right) - 1\right]\cdot\left[4\cos^2\left(\dfrac{\theta}{9}\right) - 1\right]$.

4. Let $P = e^{\theta i} = \cos(\theta) + i\sin(\theta)$. Show the following:

   (a) $P^n = \cos(n\theta) + i\sin(n\theta)$;

   (b) $1 + P + P^2 + \cdots + P^{n-1} = \dfrac{1 - P^n}{1 - P}$;

   (c) $1 + \cos(\theta) + \cos(2\theta) + \cdots + \cos[(n-1)\theta] =$

   $$\frac{[1 - \cos(\theta)][1 - \cos(n\theta)] + \sin(\theta)\sin(n\theta)}{2 - 2\cos(\theta)};$$

   (d) $\dfrac{\sin(\theta) + \sin(2\theta) + \cdots + \sin(n\theta)}{\cos(\theta) + \cos(2\theta) + \cdots + \cos(n\theta)} = \tan\left[\dfrac{(n+1)\theta}{2}\right]$.

5. Use the Euler Formula $e^{\theta i} = \cos(\theta) + i\sin(\theta)$ to express

   (a) $\cos(\theta)$ in terms of $e^{\theta i}$ and $e^{-\theta i}$;

   (b) $\sin(\theta)$ in terms of $e^{\theta i}$ and $e^{-\theta i}$.

6. For every complex number *z*, the following series converges:

$$e^{z} = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \cdots + \frac{z^n}{n!} + \cdots .$$

   Find (with some help from Problem 5) the series for:

   (a) cos(z); (b) sin(z); (c) $e^{\theta i}$; *(d) $e^{ax}\cos(bx)$, where *a* and *b* are real numbers.

7. Prove that $2\sin\left(\frac{\alpha - \beta}{2} + \frac{\pi}{4}\right)\cos\left(\frac{\alpha + \beta}{2} + \frac{\pi}{4}\right) = \sin(\beta) + \cos(\alpha)$.

8. Show that $\sin(x + y + z) = \sin(x)\cos(y)\cos(z) + \cos(x)\sin(y)\cos(z)$
   $+ \cos(x)\cos(y)\sin(z) - \sin(x)\sin(y)\sin(z)$.

9. Show that $\cos(x + y + z) = \cos(x)\cos(y)\cos(z) - \sin(x)\sin(y)\cos(z)$
   $- \sin(x)\cos(y)\sin(z) - \cos(x)\sin(y)\sin(z)$.

10. (a) With the calculator in radian mode press RS SYMBOLIC DA m-OK to get into the differentiation dialog box. Enter 'SIN(X)' in the EXPR: box, X in the VAR: box, and make sure that "Symbolic" is chosen in the RESULT: box, then press m-OK.

    (b) Leave the result of (a) on the stack but change the mode to degrees and repeat part (a).

    (c) Why do you get different results to parts (a) and (b)?

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