

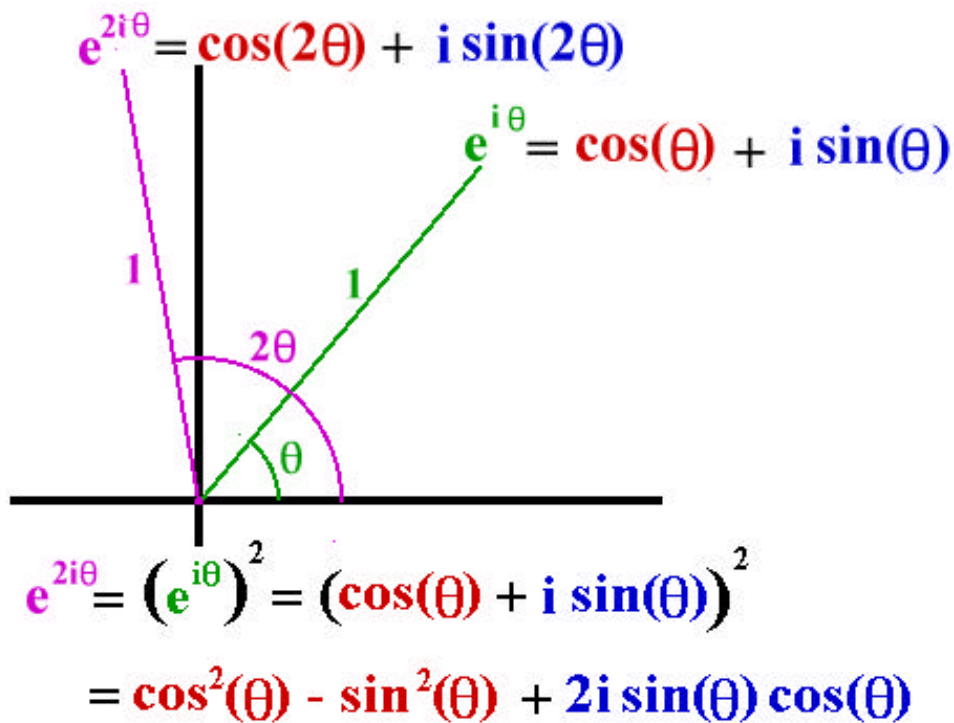
Complex Numbers and Trigonometry

by

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$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

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To

Josephine Hillman

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Preface

This treatment of trigonometry makes it easier to derive and much easier to remember key concepts; it has proved to be most helpful for further study in mathematics, science, and engineering. The original text *Functional Trigonometry* by Hillman and Alexanderson was used successfully in many high schools during the 70's and mid 80's. The predecessors of this short form were very helpful as supplements at The University of New Mexico and at Santa Clara University in various courses, especially for students who were not quite ready for calculus. This version has been augmented with examples and exercises which make use of a graphics calculator. Although the calculator chosen here is the HP 48G, the examples and problems could just as well be done on almost any graphics calculator. Be warned, however, that the HP 48G uses RPN logic, so the reader using a calculator with algebraic logic will have to make extensive changes to the keystroke sequences given in the examples of this text.

The HP 48G has three shift keys, the alpha [α] shift, the lavender left shift and the aqua right shift. In the examples in this text these shift keys will be abbreviated AS, LS, and RS respectively. The four arrow keys; up arrow, down arrow, left arrow, and right arrow; will be abbreviated UA, DA, LA, and RA respectively. Finally, menu commands will be preceded by m-. To see how this works, let's get our calculator ready for the first calculator example, which is in Section 2 of Chapter 1. That example asks us to find a quantity to one decimal place. To accomplish that, we want to set our display mode to Fix 1. Assuming our calculator is now set to the standard display, the following sequence will get it ready for Calculator Exercise 1.2.1:

RS MODES m-CHOOS DA m-OK RA 1 m-OK m-OK

For complete instruction on how to set various display modes see page 4-2 of *HP 48G Series User's Guide* which came with the calculator. In the future this publication will be referred to as *UG*.

In the exercises in this text you will at times be asked for exact values and at other times for an approximation to some number of decimal places. In the case of exact answers, a calculator should not be used, in the case of a decimal approximation, a calculator will almost always be needed. Suppose, for example, that you are asked to find the value of c and the value of θ in radians in Figure 1.

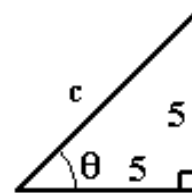


Figure 1

If you are asked for exact values, your responses should be $c = 5\sqrt{2}$ and $\theta = \pi/4$. On the other hand, if c is required to two decimal places and θ to four decimal places, your responses should be $c = 7.07$ and $\theta = 0.7854$, and you would certainly use your calculator to compute these.

Both forms of the solutions are important for different purposes. For theoretical purposes, especially for problem solving by pattern recognition, it is usually much more productive to work with exact values. On the other hand, if c is in inches, and you are to cut a piece of string of that length, the decimal approximation is certainly much more useful.

Many of the problems in the exercise sets which follow are required for understanding of concepts in later sections. These problems are distinguished by having their problem numbers in bold face type.

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An interactive version of this text can be found on the WWW at <http://www.thiel.edu/mathproject/Cnat/>