

# Complex Numbers and Trigonometry

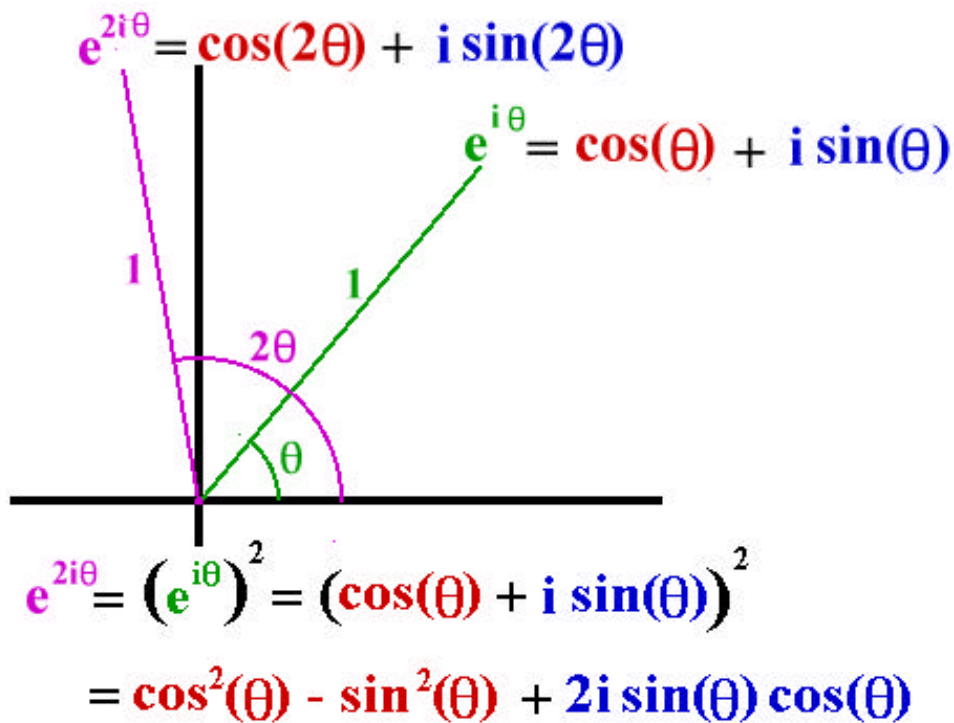
## Third Edition

by

**Abraham P. Hillman**  
University of New Mexico

**Gerald L. Alexanderson**  
Santa Clara University

**Mervin E. Newton**  
Thiel College



$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

Copyright © 2005

by

Abraham P. Hillman    Gerald L. Alexanderson    Mervin E. Newton

All rights reserved

To

Josephine Hillman

## Table of Contents

Copyright .....	i
Preface .....	iii
Chapter I .....	1
1. Rays and Segments .....	1
2. Angles and Triangles .....	2
3. Similar Triangles .....	3
4. Important Special Triangles .....	5
5. Parallel Lines and Parallelograms .....	6
Exercises for Chapter I .....	6
Chapter II .....	10
1. Polar Form .....	10
2. Terminology .....	10
Exercises for Chapter II Sections 1 and 2 .....	11
3. Negative of a Point, Subtraction, Conjugate .....	15
4. Reciprocal of a Point, Division .....	15
5. The Real and Imaginary Axes; Rectangular Form .....	16
Exercises for Chapter II Sections 3, 4, and 5 .....	18
6. Complex Numbers on the Calculator - Polar Form .....	24
7. Rectangular Form on the Calculator .....	27
Exercises for Chapter II Sections 6 and 7 .....	28
Chapter III .....	31
1. The Trigonometric Functions .....	31
Exercises for Chapter III Section 1 .....	33
2. The Inverse Trigonometric Functions. ....	37
Exercises for Chapter III Section 2 .....	38
3. Solving Triangles .....	39
Exercises for Chapter III Section 3 .....	42
4. Trigonometry on the Calculator. ....	42
Exercises for Chapter III Section 4 .....	47
Supplementary Problems .....	50
Answers and Hints for Selected Odd Numbered Problems .....	53
INDEX .....	58

## Preface

This treatment of trigonometry makes it easier to derive and much easier to remember key concepts. It has proved to be most helpful for further study in mathematics, science, and engineering. The original text *Functional Trigonometry* by Hillman and Alexanderson was used successfully in many high schools during the 70's and mid 80's. The predecessors of this short form were very helpful as supplements at The University of New Mexico and at Santa Clara University in various courses, especially for students who were not quite ready for calculus. This version has been augmented with examples and exercises which make use of a graphics calculator. The only difference between this Third Edition and the previous edition (the Revised Edition) is that this one uses the newer HP 49G+, whereas the previous edition used the HP 48GX. The Revised Edition is no longer available, but the original edition is still on the Web. A reader who wishes to see the calculator examples on the HP 48GX will find them in the original edition. Although the calculator chosen here is the HP 49G+, the examples and problems could just as well be done on almost any graphics calculator. On the HP48GII the commands will be exactly as they are on the 49G+. All the examples are given using RPN logic, so the reader using a calculator with algebraic logic will have to make extensive changes to the keystroke sequences given in the examples of this text.

The HP 49G+ has three shift keys, the yellow ALPHA shift, the green left shift and the red right shift. In the examples in this text these shift keys will be abbreviated AS, LS, and RS respectively. The four arrow keys; up arrow, down arrow, left arrow, and right arrow; will be abbreviated UA, DA, LA, and RA respectively. In some cases, especially with the calculator in RPN mode, a shift key must be held down while another key is pressed. In such cases the command will be written as LS(hold). Finally, menu commands will be preceded by the appropriate soft key, F1, F2, ..., F6. To see how this works, let's get our calculator ready for the first calculator example, which is in Section 2 of Chapter 1. The first thing we want to do is to make sure that the calculator is in RPN mode and that the menus will show up as soft keys. To do this press MODE. If the Operating Mode does not show RPN, press the +/- key. Now press F1-Flags and press UA 7 times to highlight system flag 117. If this flag is not checked, press F3-CHK. The flag should now show "Soft MENU." Now press F6-OK twice.

The example for which we are preparing asks us to find a quantity to one decimal place. To accomplish that, we want to set our display mode to Fix 1. Assuming our calculator is now set to the standard display, either of the following sequences will get it ready for Calculator Example 1.2.1:

```
MODE DA F2-CHOOS DA F6-OK RA 1 F6-OK
LS PRG NXT F4-MODES F1-FMT 1 F2-FIX
```

For complete instruction on how to set various display modes see page 1-17 of *HP 49G+ Graphing Calculator User's Guide* which came as a PDF file on a CD with the calculator. In the future this publication will be referred to as *UG*. **NOTE:** *UG* can also be found on the Web at <http://h10032.www1.hp.com/ctg/Manual/bpia5324.pdf>

In the exercises in this text you will at times be asked for exact values and at other times for an approximation to some number of decimal places. In the case of exact answers, a calculator should not be used, in the case of a decimal approximation, a calculator will almost always be needed. Suppose, for example, that you are asked to find the value of  $c$  and the value of  $\theta$  in radians in Figure 1.

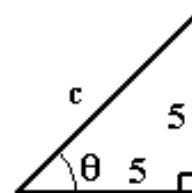


Figure 1

If you are asked for exact values, your responses should be  $c = 5\sqrt{2}$  and  $\theta = \pi/4$ . On the other hand, if  $c$  is required to two decimal places and  $\theta$  to four decimal places, your responses should be  $c = 7.07$  and  $\theta = 0.7854$  and you would certainly use your calculator to compute these.

Both forms of the solutions are important for different purposes. For theoretical purposes, especially for problem solving by pattern recognition, it is usually much more productive to work with exact values. On the other hand, if  $c$  is in inches, and you are to cut a piece of string of that length, the decimal approximation is certainly much more useful.

One final note about calculators. The authors recognize that efficient use of technology is a necessary part of mathematical training, especially for students going into science and engineering. On the other hand, knowing which button to push on the calculator is not a substitute for understanding concepts. In Chapter I, where we are simply reviewing known concepts and terminology, the calculator instructions and examples are integrated throughout the chapter. In Chapters II and III, however, where the concepts are presumed new to the reader, calculator usage is postponed to the end of the chapters. The reader is encouraged to resist starting the calculator sections until the concepts in the early part of those chapters have been thoroughly mastered. Those readers who prefer not to deal with the calculator can omit all of the calculator examples in Chapter I and the sections on calculator usage at the end of Chapters II and III without loss of essential concepts. It should be noted, however, that many of the calculator examples and exercises do provide additional illustration and clarification of those concepts.

Many of the problems in the exercise sets which follow are required for understanding of concepts in later sections. These problems are distinguished by having their problem numbers in bold face type.

The authors wish to thank, Josephine Hillman, Sara Franco Newton, Dane Haskings (Thiel class of 2006), Kara McDowell (Thiel class of 2001), Amanda McKeehan (Thiel class of 2004), Andrew Murrin (Thiel postgraduate student), Nicole Volchko (Thiel class of 2000), Sean Weaver (Thiel class of 2004), Rebekah Williams (Thiel class of 2004), and Jan Willman (Thiel class of 2004), for their help in proof reading this and the previous editions of this text. Thanks also to Michelle Porada (Thiel class of 2000) and Jonathan Manko (Thiel class of 2004) for their help in preparing this work for the WWW.

The authors would also like to thank Giuseppe Cammarata, who used this text in one of his courses at the Liceo Scientifico Benedetto Croce in Palermo, Italy and Dr. Karl Oman of Thiel College, both of whom made many valuable suggestions for this and/or previous edition of this text.

An interactive version of this text can be found on the WWW at

<http://www.thiel.edu/mathproject/Cnat/>