

Calculator Lesson 24

Function Agreement

NOTE: The figures in this lesson were drawn with an HP-49g+, but the graphs will look basically the same on HP-50g except that the enunciators don't show at the top of the screen. This is the difference in the graphics between the 50 and earlier models that makes the display area of the graphics a little bigger on the 50.

If two continuous functions f and g have the property that $f(x_0) = g(x_0)$ the functions are said to **agree at x_0** . As a measure of how closely the two function agree in a neighborhood of x_0 we will look at the rate at which $|f(x) - g(x)|$ approaches zero as x approaches x_0 . Our measuring tool will be the functions $|x|, |x|^2, |x|^3, \dots$. Note that for $0 < |x| < 1$, $|x| > |x|^2 > |x|^3 > \dots > 0$, so we can see that the larger n is, the faster $|x|^n$ approaches zero. We will say that the agreement between f and g is of order n if $|f(x) - g(x)|$ approaches zero as x approaches x_0 at least as fast as $|x|^n$ approaches zero as x approaches zero, but not as fast as $|x|^{n+1}$ approaches zero. We will do the measuring graphically using a procedure base on the text *Calculus* (now out of print) by Thomas P. Dick and Charles M. Patton. To see how it works graph $\{|x|^2, |x|^3, |x|^4\}$ with H-View from -0.65 to 0.65 and V-View from -0.39 to 0.4. Be sure that Simult is not checked so that the curves are drawn one at a time and we can tell which is which (recall that the curves will be drawn in order from left to right). We see that $|x|^2$ is the top graph, $|x|^3$ is in the middle and $|x|^4$ is lowest. See Figure 1. Now press F1-ZOOM then F1-ZFACT. Set H-Factor to 10, V-Factor to 1000, check Recenter on Cursor, then press F6-OK. Now press F3-ZIN. We see that the side of $|x|^2$ have become much steeper, $|x|^3$ has held its shape, and $|x|^4$ has become much flatter. See Figure 2.

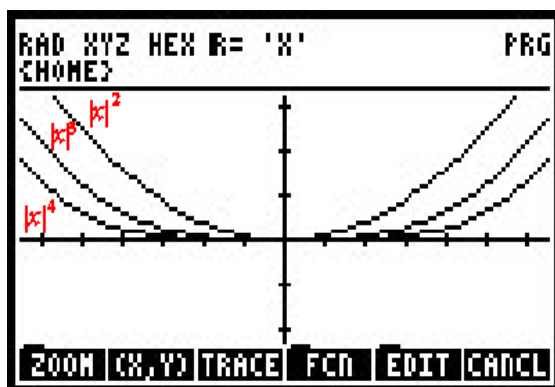


Figure 1

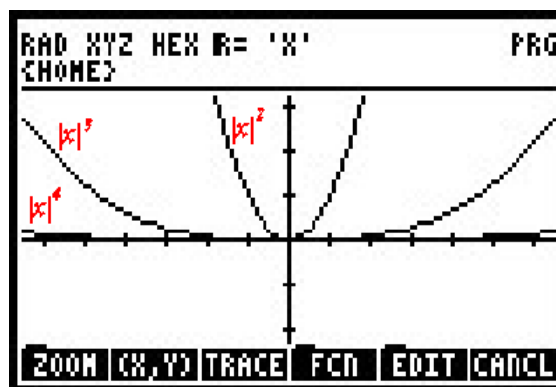


Figure 2

This is typical behavior. If we zoom in with H-Factor set to 10 and V-Factor set to 10^n , $|x|^n$ will keep its shape, $|x|^m$ will become steeper if $m < n$ and flatter if $m > n$.

We are now in a position to test the order of agreement of two functions. Let us find the order of agreement of $f(x) = \cos(x)$ and $g(x) = 1 - x^2$ at $x = 0$. Graph $|\cos(x)^2 - (1 - x^2)|$ with H-View -0.65 to 0.65 and V-View -0.39 to 0.4. See Figure 3. Now zoom in with H-Factor set to 10 and V-Factor set to 10. We see that the graph has become very flat, which tells us that the agreement must be greater than 1. See Figure 4. Press F1-ZOOM > NXT > NXT > F4-ZLAST to return to our original graph and zoom in again with V-Factor set to 100. This time the curve holds its shape, so we believe the degree of agreement is order 2. See Figure 5. We verify by pressing zooming in again with the same setting and get the same result. See Figure 6. As a final test we zoom in once more with V-Factor set to 1000. This time the graph becomes much steeper, which tells us that $n = 3$ is too big, the order of agreement is 2. See Figure 7.

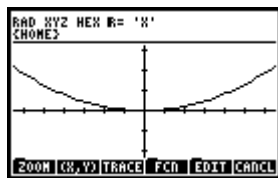


Figure 3

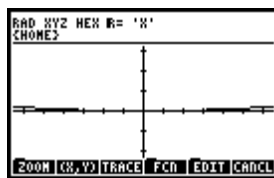


Figure 4

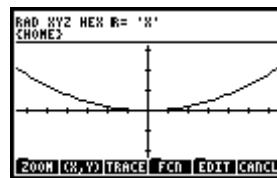


Figure 5

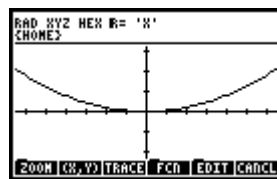


Figure 6

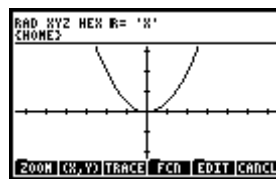


Figure 7

Our next example will be to try to find the order of agreement of $f(x) = \ln(x)$ and $g(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 3x - \frac{11}{6}$ at $x = 1$. Graph $\left| \ln(x) - \left(\frac{1}{3}x^3 - \frac{3}{2}x^2 + 3x - \frac{11}{6} \right) \right|$ with H-View $0.35 = 1 - 0.65$ to $1.65 = 1 + 0.65$ (the idea here is to have an H-View of length 1.3 centered at the point of agreement) and V-View -0.39 to 0.4. The curve we get is almost flat. See Figure 8. To give it more shape, we will reduce the V-View by a factor of 10 giving us -0.039 to 0.04 and draw the graph again. This gives us a curve with which we can work, see Figure 9, so we now try zooming in with a V-Factor of 10. This gives us a curve that is so completely flat it vanishes into the x -axis. See Figure 10. This suggests the order of agreement is somewhat greater than 1, so we will try 3 next. Perform a ZLAST then zoom in again with a V-Factor of 1000. See Figure 11. The curve has still flattened considerably, so we return to our original graph and zoom in again with V-Factor of 10000. See Figure 12. This time the curve has maintained about the same shape. Compare Figures 12 and 9. Another zoom with this V-Factor verifies that the order of agreement seems to be 4. See Figure 13. As a final check we zoom in with a V-Factor of 100000 and find that the curve does become much steeper, so the order of agreement is 4. See Figure 14.

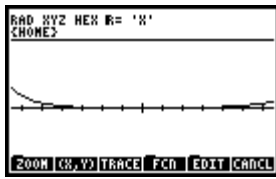


Figure 8

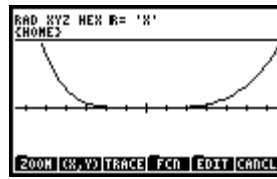


Figure 9

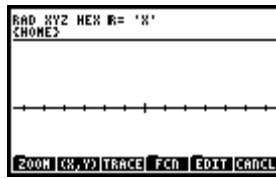


Figure 10

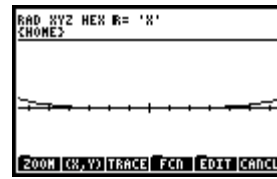


Figure 11

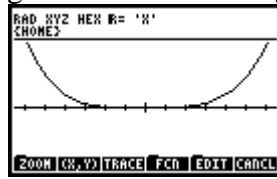


Figure 12

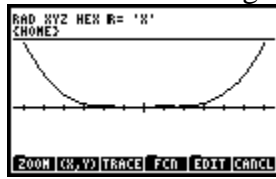


Figure 13

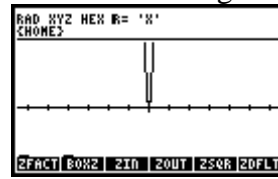


Figure 14

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