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## Calculator Lesson 20

### Taylor Polynomials

The calculator has three functions to find Taylor polynomial. We will discuss two of them in this lesson, TAYL0 and TAYLR. They are both in LS CALC F2-LIMIT and they both produce Maclaren polynomials, that is, the Taylor polynomials about  $x = 0$ . TAYL0 produces a Maclaren polynomial of degree 4 with respect to the default variable. TAYLR produces a Maclaren polynomial of any user specified order with respect to any user specified variable. It should be noted, however, that HP defines the order of the polynomial as the difference between the highest and lowest exponents on the variable, not necessarily the degree of the polynomial. For example, if we put 'SIN(X)' on the stack and press F4-TAYL0, we get the result

$$\frac{1}{120} X^5 + \frac{-1}{6} X^3 + X.$$

We can get the same result by putting 'SIN(X)' on level 3, 'X' on level 2 and 4 on level 1 and pressing F5-TAYLR. The difference is that with TAYLR we can find the polynomial with respect to any variable we enter on level 2 and to any order we enter in level 1.

To get a Taylor polynomial for a function  $f(x)$  about some point  $x = a \neq 0$  we need to do a little trick. First use TAYL0 or TAYLR to find the Maclaren polynomial for  $f(x + a)$ , then substitute  $(x - a)$  for  $x$  in the result. For example, suppose we want the 4 order Taylor polynomial for  $\sin(x)$  about the point  $x = \frac{\pi}{3}$ . Put  $\sin\left(x + \frac{\pi}{3}\right)$  on the stack and press F4-TAYL0. You will see the result

$$\frac{\sqrt{3}}{48} x^4 + \frac{-1}{12} x^3 - \frac{\sqrt{3}}{4} x^2 + \frac{1}{2} x + \frac{\sqrt{3}}{2}.$$

We must now replace  $x$  with  $\left(x - \frac{\pi}{3}\right)$ . To do this on the calculator put 'X = X -  $\pi/3$ ' on level 1 of the stack and press RS ALG NXT F2-SUBST and the correct solution

$$\frac{\sqrt{3}}{48} \left(x - \frac{\pi}{3}\right)^4 + \frac{-1}{12} \left(x - \frac{\pi}{3}\right)^3 - \frac{\sqrt{3}}{4} \left(x - \frac{\pi}{3}\right)^2 + \frac{1}{2} \left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$$

will be on the stack.

As another example suppose we want the order 3 Taylor polynomial of  $\ln(t)$  about the point  $t = 1$ . Press LS CALC F2-LIMIT to get back into the correct directory. Now put 'ln(t + 1)' on level 3 of the stack 't' on level 2 and 3 on level 1 then press

F5-TAYLR. Now enter ' $t = t - 1$ ' and press RS ALG NXT F2-SUBST and the correct solution:

$$\frac{1}{3}(t-1)^3 + \frac{-1}{2}(t-1)^2 + t - 1$$

will show on the stack.

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