Return to List of Lessons

Calculator Lesson 5

Power and Exponential Functions

Power and exponential functions are closely related, but different. The expression $x^{p/q} = \sqrt[q]{x^p} = \left(\sqrt[q]{x}\right)^p$ where p/q is a reduced fraction is defined whenever it can be computed to give a real value for an answer. Thus, $(-8)^{5/3} = \left(\sqrt[3]{-8}\right)^5 = (-2)^5 = -32$ is well defined if thought of as a special case of the power function $x^{5/3}$ with $x = -8$. Most introductory calculus texts give only an intuitive definition of the exponential function a^x and the parameter is restricted to $a > 0$ and $a \neq 1$. A proper definition of a^x must wait until we have mastered certain concepts from calculus. In a complex analysis course the definition is extended to include $a < 0$, but that is beyond the scope of Calculus I. According to that definition, if $a < 0$ and x is not an integer, then a^x is a complex number. We are somewhat familiar with this since we know that $(-1)^{.5} = \sqrt{-1} = i$. Thus, if we think of $(-8)^{5/3}$ as a special case of the exponential function $(-8)^x$ with $x = 5/3$, the value should be a complex number. Thus, calculator manufactures must make a choice of how to interpret an expression such and $(-8)^{5/3}$, as a special case of a power function or as a special case of an exponential function. Hewlett Packard has chosen to interpret it as a special case of the exponential function, so you will get a complex answer. If we wish to evaluate $x^{p/q}$ as a power function with x negative and q odd, we must use the $\sqrt[x]{y}$ command. For a particular value, such as $(-8)^{5/3}$, enter 8 +/- ENTER 5 Y^X 3 RS $\sqrt[x]{y}$ and you will see the answer -32. To create the function $G(x) = \sqrt[3]{x^5}$ get into the equation writer and enter the following sequence:

ALPHA G LS ( ) X RA RS = 3 RS

$\sqrt[x]{y}$ X Y^X 5 ENTER

You can now define and use the function as we learned in Lesson 2.

Return to List of Lessons